1P1-2

Relationship between Displacement Current and Magnetic Field from the Viewpoint of Roentgen Current in Dielectric and Piezoelectric Materials

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1. Introduction

There is ongoing debate as to whether an electric displacement current $\boldsymbol{J}_{\mathrm{d}}$ can generate a magnetic field $\boldsymbol{H}$ or not[1]. Although Feynman points out the ambiguity of whether $\boldsymbol{J}_{\mathrm{d}}$ or a conduction current $\boldsymbol{J}$ generates $\boldsymbol{H}$[2], some researchers assert the followings: (1) The electric displacement current derived from a longitudinal electric field $\boldsymbol{E}_{(\mathrm{L})}$, $\boldsymbol{J}_{\mathrm{d(L)}} = \varepsilon \frac{\partial \boldsymbol{E}_{(\mathrm{L})}}{\partial t}$, cannot generate $\boldsymbol{H}$, since $\boldsymbol{E}_{(\mathrm{L})}$ is irrotational (conservative) as $\boldsymbol{\nabla} \times \boldsymbol{E}_{(\mathrm{L})} = \boldsymbol{0}$. (For example, a point charge generates an irrotational electric field, and so does its superposition, which cannot generate $\boldsymbol{H}$); (2) Only $\boldsymbol{J}$ can be regarded as a source of $\boldsymbol{H}$, and there is no cause-effect relations between $\boldsymbol{J}_{\mathrm{d}}$ and $\boldsymbol{H}$. (For example, $\boldsymbol{H}$ in a capacitor is caused by $\boldsymbol{J}$, not by $\boldsymbol{J}_{\mathrm{d}}$.)

However, the above assertions have some problems. In this study, the disproof against the above (1) and (2) is discussed by considering a dielectric or piezoelectric material that moves with a constant velocity relatively against an (inhomogeneous) electric field, which accompanies a Roentgen current $\boldsymbol{J}_{\mathrm{R}}$ as well as $\boldsymbol{J}_{\mathrm{d}}$. The consideration is based on Einstein's special relativity in inertial frames of reference.

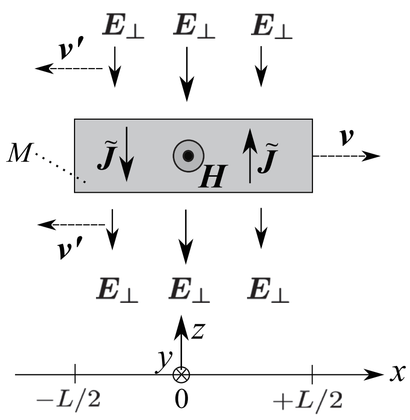

Fig. 1: Relative movement between dielectric or piezoelectric body M (moving with velocity $\boldsymbol{v}$) and spatial distribution of electric field $\boldsymbol{E}_{\perp}$ (moving with velocity $\boldsymbol{v}'$). $\tilde{\boldsymbol{J}} = \boldsymbol{J}_{\mathrm{d}} + \boldsymbol{J}_{\mathrm{R}}$, where $\boldsymbol{J}_{\mathrm{d}}$ is a displacement current in a narrow sense, and $\boldsymbol{J}_{\mathrm{R}}$ is a Roentgen current.

2. Generation of magnetic field

Figure 1 shows relative movement between an inhomogeneous spatial distribution of electric field $\boldsymbol{E}_{\perp}$ and a dielectric or piezoelectric body M. Two relative cases are considered: (A) The velocity of M is $\boldsymbol{v}$, while $\boldsymbol{E}_{\perp}$ is stationary; (B) The velocity of $\boldsymbol{E}_{\perp}$ is $\boldsymbol{v}'$ that is inverse of $\boldsymbol{v}$ in the case of (A), while M is stationary. Here $\boldsymbol{E}_{\perp}$ is the component of an external field $\boldsymbol{E}$ perpendicular to $\boldsymbol{v}$:

$$\boldsymbol{E} = \boldsymbol{E}_{\perp} + \boldsymbol{E}_{\parallel} \qquad (\boldsymbol{E}_{\perp} \perp \boldsymbol{v}), \tag{1}$$

and we assume that $\boldsymbol{E}$ is generated by superposition of point charges; that is,

$$\boldsymbol{\nabla} \times \boldsymbol{E} = \boldsymbol{0}. \tag{2}$$

In the case of (A), the movement of M with $\boldsymbol{v}$ in the environment of an electric flux density $\boldsymbol{D}$ caused by $\boldsymbol{E}$ generates the following magnetic field $\boldsymbol{H}$ in M:

$$\boldsymbol{H} = -\boldsymbol{v} \times \boldsymbol{D}, \tag{3}$$

due to Einstein's special relativity ($|\boldsymbol{v}| \ll c$ (light speed)), which accompanies a Roentgen current $\boldsymbol{J}_{\mathrm{R}}$:

$$\boldsymbol{J}_{\mathrm{R}} = -\boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{D}) = (\boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{D}, \tag{4}$$

where $\boldsymbol{\nabla} \cdot \boldsymbol{D} = 0$ and $\boldsymbol{\nabla} \cdot \boldsymbol{v} = 0$ are assumed in an inertia system.

(Roentgen's original experiment was performed in a rotational (non-inertia) system[3], as well as Eichenwald's experiment and Wilson's one that followed, to which the special relativity cannot be applied[4]. There has been some confusion and misunderstanding on this point.)

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In addition, in a material with piezoelectricity, although $\boldsymbol{E}$ causes static stress and strain distribution in M, the response of $\boldsymbol{D}$ to $\boldsymbol{v}$ is the same as in the case of non-piezoelectric materials.

In general, from eqs. (1) and (2),

$$\nabla \times \boldsymbol{E}_{\perp} = -\nabla \times \boldsymbol{E}_{\parallel} \neq \boldsymbol{0}, \tag{5}$$

unless $\nabla \times \boldsymbol{E}_{\parallel} = \boldsymbol{0}$; then $\boldsymbol{E}_{\perp}$ does not contribute to the cancel of magnetic field, even though $\nabla \times \boldsymbol{E} = \boldsymbol{0}$. The existence of $\boldsymbol{v}$ causes $\boldsymbol{H}$, while $\boldsymbol{H}$ cannot cause $\boldsymbol{v}$, regarding the cause-effect relationship.

The case of (B) is only a relative replica of the case of (A); therefore, the effect of the former is exactly the same as that of the latter. The time variation of $\boldsymbol{D}$ in M accompanied by $\boldsymbol{v}'$, $\frac{\partial \boldsymbol{D}}{\partial t}(= \boldsymbol{J}_{\mathrm{d}})$, generates $\boldsymbol{H}$, while $\boldsymbol{H}$ cannot cause $\boldsymbol{v}'$, and the cause-effect relationship is clear.

Ampere's law satisfied in M can be expressed as

$$\nabla \times \boldsymbol{H} = \tilde{\boldsymbol{J}} = \boldsymbol{J}_{\mathrm{d}} + \boldsymbol{J}_{\mathrm{R}} = \frac{\partial \boldsymbol{D}}{\partial t} - \nabla \times (\boldsymbol{v} \times \boldsymbol{D}). \tag{6}$$

3. Discussion on retarded potentials

In the configuration shown in Fig. 1, the movement of $\boldsymbol{E}_{\perp}$ with velocity $\boldsymbol{v}'$ in the case of (B) is actually originated from the movement of external point charges (charge density ρ), which implies the existence of a conduction current $\boldsymbol{J} = \rho \boldsymbol{v}'$ there. Therefore, the following interpretation is also possible: (1) $\boldsymbol{J}$ outside M is a primary source for $\boldsymbol{H}$ in M; (2) The effect of $\boldsymbol{J}$ propagates with the finite light velocity according to the special relativity, and generates $\boldsymbol{J}_{\mathrm{d}}$ in M in a retarded manner as a secondary source of $\boldsymbol{H}$.

The difference between the primary and secondary sources appears as

$$\nabla \times \boldsymbol{H} \neq \boldsymbol{J}, \quad \nabla \times \boldsymbol{H} = \boldsymbol{J}_{\mathrm{d}}, \tag{7}$$

in M, and only $\boldsymbol{J}_{\mathrm{d}}$ should be regarded as the source of $\boldsymbol{H}$ from Faraday-Maxwell's viewpoint of near field interaction. The effect of distant $\boldsymbol{J}$ propagates as retarded potentials: the retardation of both of a vector and a scalar potential, $\boldsymbol{A}$ and ϕ, respectively.

The $\boldsymbol{A}$ generated by $\boldsymbol{J}$ outside M satisfies eq. (8) at a position in M, where c is observed:

$$\nabla^2 \boldsymbol{A} = \frac{1}{c^2} \frac{\partial^2 \boldsymbol{A}}{\partial t^2}. \tag{8}$$

In eq. (8), Lorentz's gauge condition given by eq. (9) is assumed, which guarantees the propagation of both potentials with c:

$$\nabla \cdot \boldsymbol{A} = -\frac{1}{c^2} \frac{\partial \phi}{\partial t} \tag{9}$$

By subtracting $\nabla(\nabla \cdot \boldsymbol{A})$ from both sides of eq. (8),

$$-\nabla^2 \boldsymbol{A} + \nabla(\nabla \cdot \boldsymbol{A}) = -\frac{1}{c^2} \frac{\partial^2 \boldsymbol{A}}{\partial t^2} + \nabla(\nabla \cdot \boldsymbol{A}). \tag{10}$$

The left-hand side of eq. (10) equals

$$-\nabla^2 \boldsymbol{A} + \nabla(\nabla \cdot \boldsymbol{A}) = \nabla \times (\nabla \times \boldsymbol{A}) = \mu \nabla \times \boldsymbol{H}. \tag{11}$$

The first term in the right-hand side of eq. (10) is

$$-\frac{1}{c^2} \frac{\partial^2 \boldsymbol{A}}{\partial t^2} = -\frac{1}{c^2} \frac{\partial^2 (\boldsymbol{A}_{(\mathrm{L})} + \boldsymbol{A}_{(\mathrm{T})})}{\partial t^2}, \tag{12}$$

where $\nabla \times \boldsymbol{A}_{(\mathrm{L})} = \boldsymbol{0}$ (longitudinal) and $\nabla \cdot \boldsymbol{A}_{(\mathrm{T})} = 0$ (transverse). Since $\boldsymbol{E}$ can be decomposed into

$$\boldsymbol{E}_{(\mathrm{L})} + \boldsymbol{E}_{(\mathrm{T})} = \left(-\nabla \phi - \frac{\partial \boldsymbol{A}_{(\mathrm{L})}}{\partial t} \right) + \left(-\frac{\partial \boldsymbol{A}_{(\mathrm{T})}}{\partial t} \right), \tag{13}$$

the second term in the right-hand side of eq. (10) is

$$\nabla(\nabla \cdot \boldsymbol{A}) = \nabla \left(\frac{-1}{c^2} \frac{\partial \phi}{\partial t} \right) = \frac{1}{c^2} \frac{\partial}{\partial t} \left(\boldsymbol{E}_{(\mathrm{L})} + \frac{\partial \boldsymbol{A}_{(\mathrm{L})}}{\partial t} \right). \tag{14}$$

By summing up eqs. (12) and (14) with $\varepsilon \mu = 1/c^2$, the right-hand side of eq. (10) is turned into

$$\frac{1}{c^2} \frac{\partial}{\partial t} \left(\boldsymbol{E}_{(\mathrm{L})} + \boldsymbol{E}_{(\mathrm{T})} \right) = \mu (\boldsymbol{J}_{\mathrm{d(L)}} + \boldsymbol{J}_{\mathrm{d(T)}}). \tag{15}$$

Finally, eq. (10), using eqs. (11) and (15), leads to

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}_{\mathrm{d(L)}} + \boldsymbol{J}_{\mathrm{d(T)}} = \boldsymbol{J}_{\mathrm{d}}. \tag{16}$$

Both of $\boldsymbol{J}_{\mathrm{d(L)}}$ and $\boldsymbol{J}_{\mathrm{d(T)}}$ act as secondary sources of $\boldsymbol{H}$, and there is no reason to exclude $\boldsymbol{J}_{\mathrm{d(L)}}$ from the viewpoint of special relativity.

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