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2-D Finite Difference-Time Domain Simulation of Moving Multipole Sources

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1. Introduction

As the speed of trains and airplanes increases, the noise prediction from moving vehicle is required. Some works have been reported that deal with moving omni-directional sources [1] or dipole sources [2]. However, it is necessary to consider more complex directivity for real sources.

This paper focuses on multipole sources that can achieve a variety of directivity. We theoretically derive the radiated sound in the two-dimensional field when a multipole source moves and investigate its validity through numerical experiments using the 2-D finite difference-time domain (FDTD) method.

2. Theory

2.1 Moving monopole source

As shown in Fig. 1, we first consider the case where a monopole source is moving with a constant vcelocity v_S in the x-direction. A 2-D fundamental solution is given as [3]

$$p_m \approx j\frac{Q}{4}\sqrt{\frac{2}{\pi kR}}\frac{e^{j(kR-\omega t)}}{\sqrt{1-M_S\cos\theta}} \qquad (1)$$

where Q is the source amplitude, $k=\omega/c_0$ is the wave number , ω is the angular frequency of the source, $M_S = v_S/c_0$ is the Mach number of the monopole.

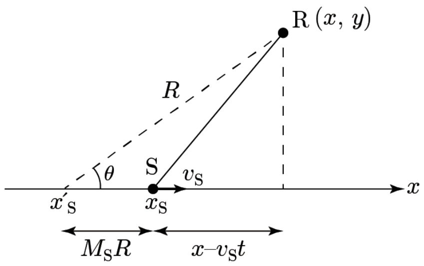

Fig. 1 Positioning of a moving monopole source and a stationary receiver.

2.2 Moving dipole sources

We next consider a dipole with positive and negative sources placed along the x-axis with an interval δ as shown in Fig. 2 (a). The radiated sound pressure from a dipole moving in the x-direction is given for $k\delta \ll 1$ and $kR \gg 1$ as [3]

$$p_x \approx \delta\frac{\partial p_m}{\partial x} \approx -\frac{k\delta Q}{4}\sqrt{\frac{2}{\pi kR}}\frac{e^{j(kR-\omega t)}}{(1-M_S\cos\theta)^{3/2}}\cos\theta \qquad (2)$$

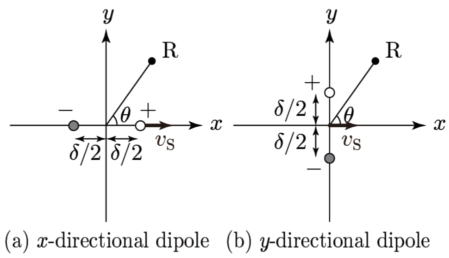

Fig. 2 Dipole sources.

Similarly, for the y-directional dipole as shown in Fig. 2 (b), the sound pressure is given as

$$p_y \approx \delta\frac{\partial p_m}{\partial y} \approx -\frac{k\delta Q}{4}\sqrt{\frac{2}{\pi kR}}\frac{e^{j(kR-\omega t)}}{(1-M_S\cos\theta)^{3/2}}\sin\theta \qquad (3)$$

2.3 Moving multipole sources

The multipole sources are composed by arranging monopole sources with spacing δ as shown in Fig. 3, with weights corresponding to the numbers in the figure. The m th-order multipole sources correspond to the m-th derivative in space. For a moving multipole source of order m in the x-direction and n in the y-direction, the sound pressure is given as

$$p_{x^m y^n} \approx j(jk)^{(m+n)}\frac{Q\delta^{(m+n)}}{4}\sqrt{\frac{2}{\pi kR}} \times \frac{e^{j(kR-\omega t)}}{(1-M_S\cos\theta)^{[2(m+n)+1]/2}}\cos^m\theta\sin^n\theta \qquad (4)$$

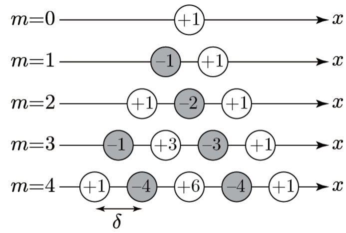

Fig. 3 x-directional multipole sources.

3. Numerical experiments

Numerical experiments are performed by the CE-FDTD (IWB) method [4,5]. Figure 4 shows a 2-D numerical model for the directivity of multipole source. The grid size is Δ=8 mm, time step is Δt=23.53 μs, and sound speed is $c_0 = 340$ m/s, so the Courant number χ is 1. The boundary condition is Mur's first order absorbing boundary. S is the center of the multipole source and R is the receiver located on a circle with a radius of 10 m. A 20-cycles sinusoidal burst was emitted from the source.

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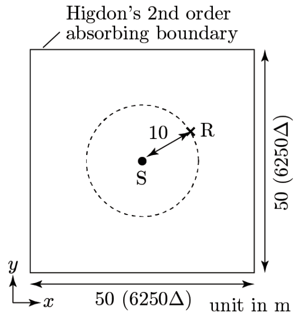

Fig. 4 Numerical model for multipole sources.

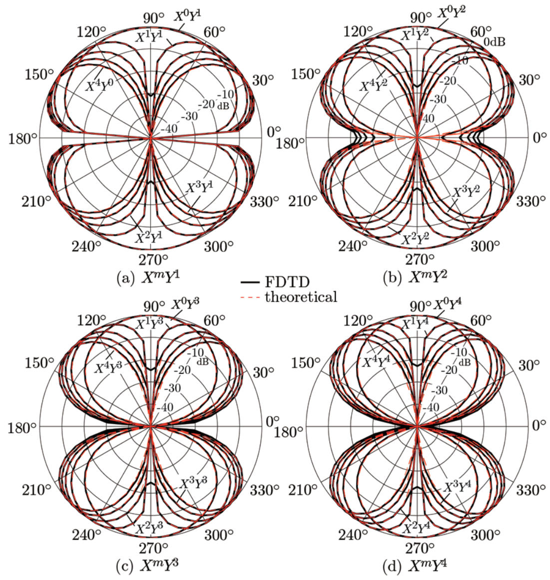

Fig. 5 Directivity of stationary multipole sources (f =500 Hz, m, $n \leq 4$).

Fig. 5 shows the directivity of stationary multipole sources of order m and n up to 4th order. The source frequency was 500 Hz and the source spacing is δ = 0.08 m ($k\delta$ = 0.739). The calculation is performed with good accuracy even when the order is increased. Fig. 6 shows the RMS error versus order m, n at source frequencies of 500, 1000, and 2000 Hz. If the order is 3 or less, the error is almost within 1%, and the directivity is well realized.

Figure 7 shows the directivity of multipole sources for source velocities $M_S = 0 \sim 0.4$, with order m, n = 0, 1. As the source speed is increased, the front-to-back ratio increases with the source velocity, and the beam width narrows. In particular, the effect of the movement is particularly pronounced when the differential direction (x-direction) coincides with the moving direction of the source. For y-directional multipole sources, the beam is closer to the moving direction as the source speed increases. Figure 8 shows the directivity of multipole sources for source speed M_S = 0, 0.2, 0.4, and orders 0, 2, and 4. The beam width in the x-direction narrows with the source velocity, and the front-back ratio increases.

These results indicate that this method is effective for the analysis of moving multipole sources.

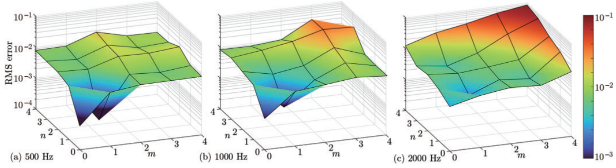

Fig. 6 RMS error against order of spatial differentiation (m, n) for stationary multipole sources.

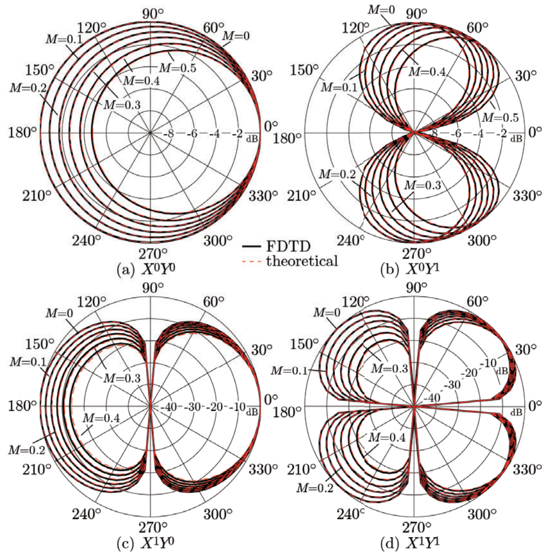

Fig. 7 Directivity of moving multipole sources (m, n = 0, 1, f = 500 Hz).

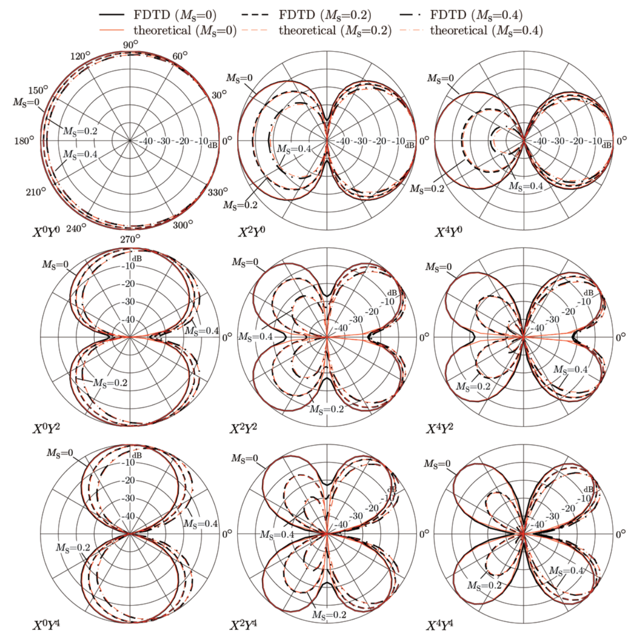

Fig. 8 Directivity of moving multipole sources.

References

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