Ultrasonic wave propagation in a sphere sandwiched between blocks

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1. Introduction

Various research has been conducted on acoustic materials and metamaterials. Some of them aim to realize specific acoustic or vibration characteristics with a system using spheres. Lydon et al.¹⁾ investigated nonlinear resonance phenomena in a one-dimensional granular chain and showed that energy was transmitted via multiple modes. It is expected that wave propagation in structures with spheres can be applied to acoustic devices such as tunable filters, and characterization of compacted powders. In this paper, we consider a structure of a sphere sandwiched by solid blocks and investigate the effects of sphere resonance on the ultrasonic propagation characteristics.

2. Numerical models and methods

As shown in **Fig. 1(a)**, a model with multiple spheres sandwiched between semi-infinite solid blocks was considered. The centers of the spheres were placed at the nodes of a square grid parallel to the *x*-*y* plane, and a longitudinal plane wave was normally incident in the negative *z* direction from the upper semi-infinite solid blocks. Since this structure was periodic in the *x*-*y* plane, a unit structure shown in **Fig. 1(b)** was extracted from the entire body, and periodic boundary condition was set on the side surfaces. Ultrasonic wave propagation behavior in this model was clarified by three-dimensional finite element analysis in the frequency domain.

A commercial finite element analysis software COMSOL Multiphysics was used to perform the numerical simulation. The material properties of stainless steel (longitudinal wave velocity 5.66 km/s, transverse wave velocity 3.12 km/s, and mass density 7.9 g/cm^3) were used to model the sphere and blocks. The diameter of the sphere was set as 5 mm, and the contact surfaces between the solid blocks and the sphere were assumed to be a circle with a radius of 25μ m, which was determined by referring to the Hertzian contact theory²). Displacement and stress components were assumed to be continuous on the contact surfaces. To suppress the reflected waves, absorbing regions were placed outside the solid



Fig. 1 (a)Schematic of a single sphere layer sandwiched by semi-infinite solid bodies. (b) Unit structure extracted from (a).



Fig. 2 Schematic of the experiment.

blocks.

3. Experimental methods

To validate the numerical results, the experiment shown in **Fig. 2** was conducted. A sphere was sandwiched by piezoelectric transducers with center frequencies of 500 kHz and 1000 kHz. A Gaussian-modulated sinusoidal wave with a frequency of 600 kHz was input to the 500 kHz transducer to generate a longitudinal wave. The transmitted wave across the sphere was measured by the 1000 kHz transducer.

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4. Numerical results

The time-averaged energy flux through the red surface in Fig. 1(b) was calculated in the frequency range from 510 kHz to 540 kHz with an increment of 1 kHz. The results are shown in **Fig. 3**. In this figure, a clear peak appears at 527 kHz. At this peak frequency, the distribution of the displacement in the z direction near the sphere is shown in **Fig. 4**. The displacement of the incident wave. The sphere appears to show an extensional vibration in the z direction.

To discuss the above result, the natural vibration modes of the sphere were analyzed. According to Saviot et al,³⁾ the natural vibrations of an elastic sphere are divided into torsional modes and spheroidal modes. As a result, a natural vibration mode, which indicates a spheroidal vibration in the *z* direction, is found at 525 kHz, as shown in **Fig. 5**.

5. Experimental results

Fig. 6 shows the measured waveform of the



Fig. 3 Normalized energy flux calculated in the single sphere layer model.



Fig. 4 Displacement distribution in the z direction at the peak frequency 527 kHz.

transmitted wave. This was analyzed by short-time Fourier transform. **Fig. 7** shows the obtained spectrogram. It is shown that only the vibrations between 500 kHz and 550 kHz, indicated by green circles, remain after 65 μ s. This seems to result from the natural vibration of the sphere shown in Fig. 5. The transmission characteristics of the longitudinal waves at the sphere are expected to change when compressive forces are applied in the structure.



Fig. 5 Displacement distribution at 525 kHz obtained by modal analysis for a sphere.



Fig. 6 The time waveform of the transmitted wave.



Fig. 7 The result of the short-time Fourier transform of the transmitted wave in Fig. 6.

References

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