

## Numerical analysis of elastic wave resonator using deformation of tube

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### 1. Introduction

Control of elastic wave propagation plays a significant role in the design of acoustic devices and metamaterials. In the field of optics, a ring resonator has been widely studied for controlling optical wave transmission.<sup>1)</sup> Recent studies reported its application to acoustic waves.<sup>2)</sup> To achieve a designated elastic wave transmission characteristic, a structure called a periodic tube-block structure is proposed in this study, based on the concept of ring resonators. The transmission characteristics of elastic waves in the structure are investigated by numerical simulations. Particularly, the effect of compressive loading on the transmission characteristics is examined in this paper.

### 2. Numerical model and method

As shown in Fig. 1, the periodic tube-block structure consists of tubes and two semi-infinite blocks. The tubes and blocks are made from isotropic linear elastic material. Tubes are periodically aligned in the  $x$  direction and sandwiched by two semi-infinite blocks. Assuming the structure to be uniform in the axial ( $z$ ) direction of the tubes, the plane-strain condition is set in the  $x$ - $y$  plane. When the structure is periodic in the  $x$  direction, a unit structure model shown in Fig. 2(a) can be obtained by setting periodic boundary conditions. The outer radius and thickness of the tubes are  $R$  and  $h$ , respectively. The width of the unit structure, i.e. the distance between

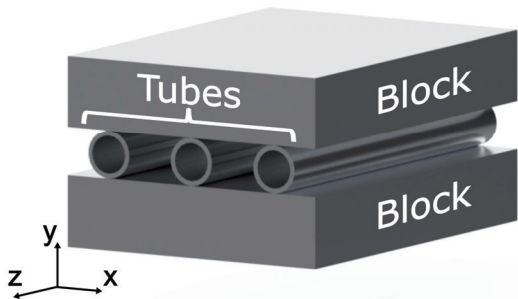


Fig. 1 Periodic tube-block structure consisting of periodically aligned tubes sandwiched by two semi-infinite blocks.

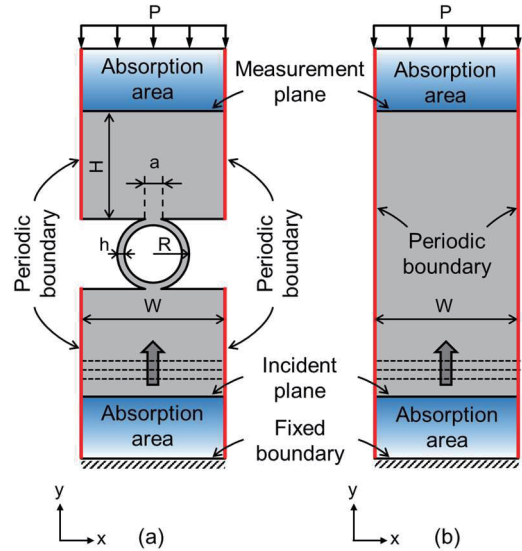


Fig. 2 (a) Numerical model of periodic tube-block structure. (b) Reference model for calculating the amplitude of the incident wave.

the center axes of adjacent tubes, is  $W$ . In this model, the tube and blocks are continuously joined with the width  $a$ .

In the frequency domain, 2D finite element (FE) method was used to analyze elastic wave propagation in the structure under prestressed conditions. Incident and measurement planes are placed at distance  $H$  from the lower and upper joints of tube and block, respectively. To suppress reflected waves, absorption areas are placed on both sides of the model. In this study, the structure is deformed by compressive loading before analyzing of elastic wave propagation. To deform the structure, one end of its surface is fixed, and a static pressure  $P$  is loaded on the other end. Under the equilibrium of the compressive situation, a longitudinal plane wave propagating in the  $y$  direction is incident from the incident plane. Fig. 2(b) is a reference model for calculating the amplitude of the incident wave, which has equivalent dimensions to Fig. 2(a).

Energy flux of elastic wave across a surface  $A$  is calculated by

$$E = - \int_A (\boldsymbol{\sigma} \dot{\mathbf{u}}) \cdot \mathbf{n} dA \quad (1)$$

where  $\boldsymbol{\sigma}$  is stress tensor,  $\dot{\mathbf{u}}$  is the time derivative of

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displacement vector, and  $\mathbf{n}$  is unit normal vector of surface  $A$ . Note that  $\boldsymbol{\sigma}$  and  $\dot{\mathbf{u}}$  is incremental quantities from the equilibrium condition. Time averaged energy flux is defined by

$$E_{\text{ave}} := \frac{1}{T_u} \int_0^{T_u} E dt \quad (2)$$

where  $T_u$  is period of the incident wave. For the two models of Figs. 2(a) and (b), the time averages of energy flux are calculated by Eq. (2) and defined as  $E_a$  and  $E_b$ , respectively. Transmission coefficient  $T$  is defined as

$$T := \frac{E_a}{E_b}. \quad (3)$$

### 3. Results and discussion

In this study, periodic tube-block structure was assumed to be made of aluminum alloy. Its density, longitudinal wave velocity, and transverse wave velocity are  $2.7 \text{ g/cm}^3$ ,  $6.40 \text{ km/s}$ , and  $3.17 \text{ km/s}$ , respectively. The frequency range of incident wave was set as  $10 \text{ kHz} - 90 \text{ kHz}$ .

#### 3.1 Effect of structure dimension

**Fig. 3** shows the frequency dependence of the transmission coefficient  $T$  for three models in the case of  $h = 1.0, 2.0, 4.0 \text{ (mm)}$  at a fixed condition  $R = 10 \text{ (mm)}$ ,  $W = 40 \text{ (mm)}$ ,  $a = 0.10 \text{ (mm)}$ ,  $H = 30 \text{ (mm)}$ , and  $P = 0 \text{ (GPa)}$ . The transmission coefficient  $T$  has many peaks at different frequencies. Peaks shown in Fig. 3 except in the vicinity of  $75 \text{ kHz}$  shifted when the tube thickness  $h$  was changed. At these peak frequencies, vibration by wave propagation concentrates on the tube. Therefore, these peaks result from tube resonance. On the other hand, at the peak frequencies in the vicinity of  $75 \text{ kHz}$ , the vibration concentrates on block surfaces. These peaks are associated with surface wave resonance.

#### 3.2 Effect of compressive lording

**Fig. 4** shows the effect of static pressure  $P$  for transmission coefficient  $T$ . The peak shift occurred

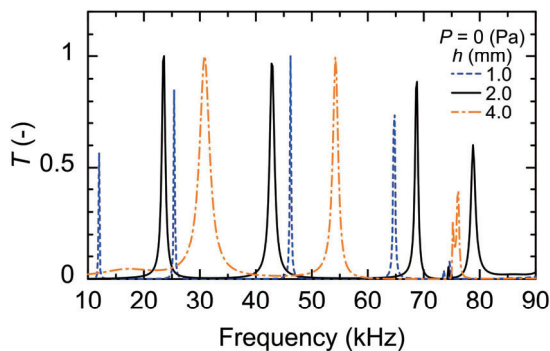


Fig. 3 The effect of the tube thickness  $h$  on the transmission coefficient.

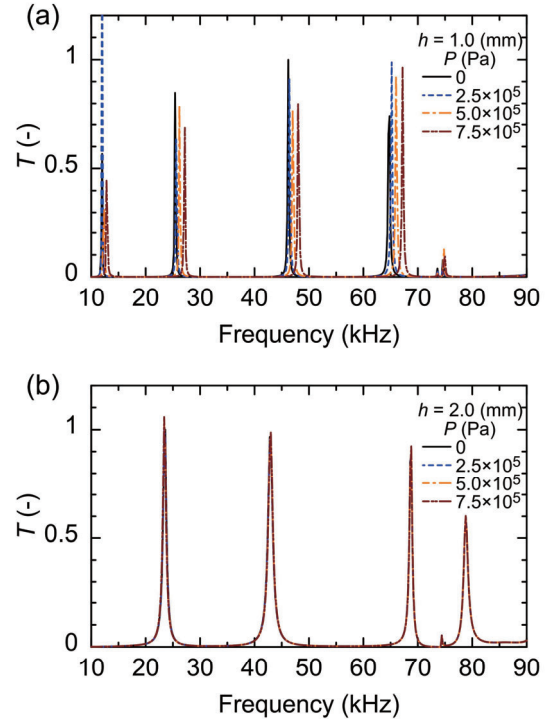


Fig. 4 The effect of static pressure  $P$  with (a)  $h = 1.0 \text{ (mm)}$  and (b)  $h = 2.0 \text{ (mm)}$ .

when the pressure was applied, and high pressure brought large shift amount. In Fig. 4(b), the tube thickness was set double to Fig. 4(a). Comparing these two figures, higher pressure was needed to occur the peak shift when the tube was thick. In general, a thicker tube shows smaller deformation at the same pressure. Therefore, the peak shift is closely associated with the tube deformation by static pressure.

These results shows that the transmission characteristic of the periodic tube-block structure is tunable by using tube deformation by static pressure.

### 4. Conclusion

This study proposed the periodic tube-block structure, and its transmission characteristics of elastic waves have been investigated. It has been shown that the transmission characteristic depends on the dimension of structure and deformation of tube. Peak frequencies have shifted when the tube thickness has changed and when the tube has been deformed by static loading.

### References

- 1) D. Rabus, *Integrated Ring Resonators The Compendium* (Springer Berlin, 2007)
- 2) W. Robertson, C. Vazquez, A. Laverde, A. Wassenberg, C. Olson, and J. Lopez, *AIP Adv.* **12**. 015006 (2022)