# Characteristics of dividers and multipliers in measurement of crystal Qs by using phase noise

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# 1. Abstract

The QCM (Quartz Crystal Microbalance) has the advantage of easily evaluating a wide variety of objects[1-3]. While most evaluation methods involve observing changes in the QCM's resonant frequency, we focused on changes in the Q factor. Furthermore, we have proposed a phase noise measurement method to measure the change in Q factor. The advantage of this method is that the Q factor can be obtained without stopping oscillation, as opposed to the QCM-D measurement system that measures the D value, which is inversely related to the Q value, by stopping oscillation.

Signal source analyzer used in the measurement have a measurement range of signals above 10 MHz, but QCMs with resonance frequencies below 10 MHz, such as 5 MHz like the QCM used in this study, are also commonly used. In this paper, experiments are conducted on the use of frequency dividers and multipliers as frequency conversion methods.

### 2. Calculation of Q-value

The Leeson model is used to calculate the Q factor from the phase noise characteristics. According to the Leeson model, a phase noise  $S_{\phi}$  can be written as:

$$S_{\phi}(f) = \alpha \left(\frac{\nu_0}{2Q_L}\right)^2 \cdot f^{-3} + \beta \left(\frac{\nu_0}{2Q_L}\right)^2 \cdot f^{-2} + \alpha f^{-1} + \beta$$

where f is the offset frequency,  $v_0$  is the nominal frequency,  $Q_L$  is the Q factor,  $\alpha$  is the 1/f noise level constant in open loop,  $\beta$  is the noise floor level [4].

The procedure is as follows. First, the phase noise characteristics are measured, and values that deviate significantly from the ideal characteristics are removed from the data as noise. Noise is generally generated like a thorn in the phase noise characteristic as shown in **Fig. 1**.



Fig. 1 Noise isolated phase noise characteristics.

Next, in the range where f is sufficiently large, i.e.,  $f = 1 \sim 100$  kHz in this case, the previous two terms are ignored and  $\alpha$  is calculated. Finally, in the range where f is sufficiently small, i.e.,  $f=1 \sim 10$ Hz in this case, the Q factor is calculated by ignoring the back three terms and substituting the  $\alpha$  obtained in the previous step.

To obtain the reference Q-factor, the conventional method of obtaining it from the frequency response is used. The reference Q factor  $Q_r$  can be written as:

$$Q_r = \frac{\omega_0}{\Delta \omega}$$

where  $\omega_0$  is frequency at which characteristic values (impedance, etc.) are maximum,  $\Delta \omega$  is Frequency range where characteristic value is  $1/\sqrt{2}$ .

## 3. Experimental system

**Fig. 2** shows the CMOS oscillation circuit for oscillating the QCM and reference crystal. **Fig. 3** is a block diagram using the oscillation circuit of Fig. 2, plus a mixer for mixing with the reference oscillation circuit to convert the 5 MHz resonance frequency of the QCM to 10 MHz or higher, the measurement range of the signal source analyzer. The reference oscillator is 10 MHz, and the mixer output is around 15 MHz Here, we use a bandpass filter (BPF) to remove 5 MHz. Measurements were also taken when a frequency divider or a multiplier was inserted between the LPF and the mixer. In order to divert the BPF and signal source analyzer settings, the reference oscillator frequency is changed to 5 MHz in Fig. 3.



Fig. 2 Circuit configuration of CMOS oscillation circuit.



Fig. 3 A block diagram to measure phase noises of QCM (5 MHz) as a reference.

#### 4. Experimental results

**Fig. 4** shows the measured results of the experiment. The blue, orange, and gray lines show the phase noise for the reference, when a divider is inserted, and when a multiplier is inserted, respectively. It can be seen that the deviation is at most about 4 dBc/Hz at an offset frequency of 10 Hz.



Fig. 4 Results of phase noise with QCM devices.

Based on Fig. 4, the respective Q factors were calculated using the Leeson model: 65000 for the reference, 50000 for the divider and 91000 for the multiplier. Also, since the reference values of the

Q factor are 58000, we can calculate their relative errors: 0.11 for the reference, -0.14 for the divider and 0.57 for the multiplier

## 5. Conclusions

From our past studies, we know that the change in phase noise when a metal such as tantalum or titanium is deposited on the QCM is about 17 dBc/Hz at an offset frequency of 10 Hz. In comparison, the maximum deviation when a divider or multiplier is inserted is 4 dBc/Hz, which is not considered to be a cause of unmeasurable error. In fact, when the Q value is calculated using the Leeson equation, the relative error for the frequency divider is -0.14, which is close to the relative error of 0.11before the insertion of the frequency divider. However, the relative error of the multiplier is larger at 0.57. The reason for this is currently under investigation. However, in order to measure QCMs with low resonance frequencies such as 5 MHz with a signal source analyzer, it is necessary to convert the frequency up, and it may be difficult to use a multiplier as a method to do so. Furthermore, if conversion to a lower frequency is required, such as in IoT applications, the use of a multi-stage multiplier will be necessary, and the impact of this error can be expected to be even greater.

Therefore, the conclusion is that heterodyne detection or frequency divider is the better frequency conversion method for the current situation.

#### References

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