

A decision feedback equalizer with L1-norm regularization in underwater acoustic communication

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1. Introduction

In high data rate underwater acoustic (UWA) communication using coherent modulation methods including single carrier modulation often encounters the difficulties of the long and time-varying impulse responses. To overcome the time varying nature of UWA channels, the decision feedback equalizer (DFE) based on adaptive algorithms such as least mean square (LMS) are widely applied to the UWA channels¹⁾. The DFE equipped with a digital phase locked loop (DPLL) has demonstrated high equalization performance by closely tracking the time variation of the UWA channel. However, in environments with long-delay multipath, where sparse responses are often observed, it is known to require long filter tap lengths. Classical adaptive algorithms may encounter issues such as increased computational cost and instability in finding solutions under such conditions.

In this study, we derived a DFE based on a simple L1-norm regularized LMS (L1-LMS) algorithm and compared it with the conventional DFE based on the LMS algorithm using the equalization results of simulation data. The proposed method resulted in the formation of filter taps that are suitable for sparse UWA channel responses, and it achieved stable bit error rate (BER) performance in communication signal processing.

2. L1-norm regularized DFE

When an acoustic communication signal is transmitted from an underwater acoustic transducer and the signal is recorded at M -element receiver array, the received symbol of the m th element of the receiver channel can be given by the symbolic form as: $y_m(l) = \mathbf{h}_m^T(l)\mathbf{d}(l) + n(l)$, where the received symbol $y_m(l)$, the channel response $\mathbf{h}_m(l)$, transmitted symbol $\mathbf{d}(l)$, and the noise term $n(l)$. The purpose of the communication signal processing is to estimate the transmitted symbols from received symbols via the equalization process. The system model of the multi-channel DFE for l th symbol is expressed as follows:

$$d_{est}(l) = \sum_{m=1}^M \exp(-i\theta_m(l)) \mathbf{a}_m^H(l) \mathbf{y}_m(l) + \mathbf{b}^H(l) \mathbf{d}_{dec}(l), \quad (1)$$

where the l th equalized symbol $d_{est}(l)$, the phase tracking parameter of the DPLL $\theta_m(l)$, the

feedforward filter of the m th element of the receiver channel $\mathbf{a}_m(l)$, the vector form of the input received symbols $\mathbf{y}_m(l)$, the feedback filter $\mathbf{b}(l)$, and the decision result of the equalized symbols $\mathbf{d}_{dec}(l)$ at the decision device of the DFE. The overscript $[\]^H$ and $[\]^T$ denote the hermite transpose and transpose, respectively.

Conventional LMS algorithm updates the filter coefficients using the minimum square error (MSE) criteria¹⁾ $|E(l)|^2 = |d_{dec}(l) - d_{est}(l)|^2$ as:

$$\begin{aligned} \mathbf{a}_m(l+1) &= \mathbf{a}_m(l) + \mu_F E^*(l) \mathbf{y}_m(l) \exp(-i\theta_m(l)) \\ \mathbf{b}(l+1) &= \mathbf{b}(l) + \mu_B E^* \mathbf{d}_{est} \end{aligned} \quad (2)$$

where μ_F and μ_B are step size parameters. In the DFE using a simple MSE minimization-based update algorithm, there are issues such as the inability to converge when using a long filter taps due to the increased complexity of the model and the propagation of errors to subsequent symbol equalization, leading to burst errors when an error occurs once.

Assuming the sparse UWA channel characteristic, we introduce the L1-norm of the model parameter into the objective function as:

$$E_{L1}(l) = |E(l)|^2 + \lambda_{FF} \sum_{m=1}^M \|\mathbf{a}_m(l)\|_1 + \lambda_{FB} \|\mathbf{b}(l)\|_1, \quad (3)$$

where λ_{FF} and λ_{FB} are regularization parameters for feedforward and feedback filters, respectively. Since each filter coefficient $\mathbf{a}_m(l)$, $\mathbf{b}(l)$ and $\theta_m(l)$ are independent variables, the gradient for updating the filter coefficients can be simply derived by the partial derivatives of Eq. (3) by each filter coefficient. The phase tracking parameter of the DPLL is updated using the second order LMS algorithm¹⁾.

In this study, the received communication signal is processed as follows:

- (1) Synchronize the communication signal frame using the cross-correlation results of the synchronization codes inserted front/end of the communication signal frame.
 - (2) Using the synchronization results, compression/dilatation of the signal frame is computed, and this result is utilized for the coarse Doppler compensation in resampling at the digital down conversion from passband signals to baseband symbols.
 - (3) Apply multi-channel DFE schemes.
- Finally, the hard decision result of the DFE output is

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evaluated for the comparison of the methods in the form of the bit-error rate (BER).

3. Simulation Result

In this study, each equalization scheme was applied to a simulation dataset of UWA environment. The power-delay profile of the time-varying channel response generated by an acoustic ray-tracing simulation²⁾ is depicted in Fig. 1. In the simulation, a vertical UWA channel between the near bottom located sound source and 4-channel receiver array beneath the sea surface is assumed. The temporal fluctuation of the channel response in Fig. 1 is caused by the vertical motion of the receiver channel. This fluctuation causes the Doppler spread in channel response which are roughly estimated as ± 5 Hz. The detailed parameters for the simulation of the UWA communication are summarized in Table 1.

Here, the results of the step size parameters μ_F , μ_B , λ_{FF} and λ_{FB} in Eqs. (2) and (3) set as 0.005, 0.005, 0.05, and 0.05, respectively, are discussed. The comparison of the amplitude of the feedback filter taps of the L1-LMS and the conventional LMS based DFE using the data of SNR = 18 dB. In L1-LMS, it can be seen that the filter taps show near sparse distribution compared with the conventional LMS. In conventional LMS, it is considered that the effect of the noise is reflected to the filter tap amplitudes. The comparison of the BER performance results is shown in Fig. 3. In this figure, the results of different 28 communication signal frames are overwritten. Our L1-LMS scheme shows better results in two aspects. First, in LMS, there are data with large BER even when the EbNo (normalized SNR per information bit) is high, but in L1-LMS, it is stable regardless of the data. In LMS, when EbNo is less than 10dB, the value is almost BER ~ 0.5 , but in L1-LMS, the BER characteristics are clearly improved as EbNo is improved even in this range.

4. Conclusion

In this study, the DFE method using the L1-norm regularized LMS is proposed, and its performance is compared with conventional LMS based DFE using the simulation data of UWA communication. From the BER characteristics, it is found that L1-LMS shows higher robustness than the conventional method.

References

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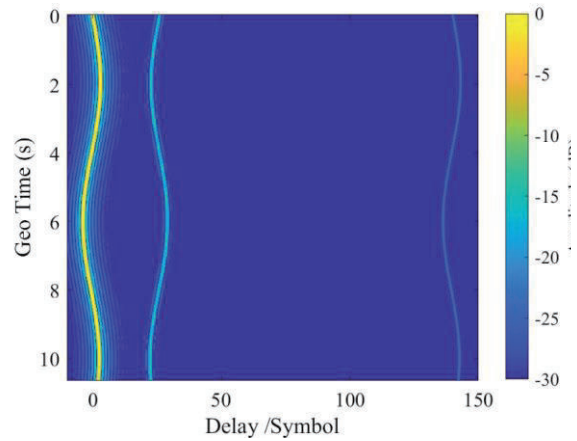


Fig. 1 An example of the time-varying channel delay-power profile of the simulation data

Table 1 Key parameters for the simulation

Parameter	Value
Carrier Freq.	20 kHz
Bandwidth	10 kHz
Modulation	Single Carrier
M-ary	4, (16 QAM)
Symbols per frame	3240 Symbols
Num. of Rx. channels	4
Rx Array	0.1x0.1m square

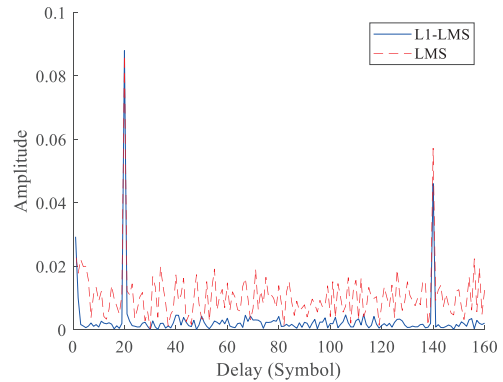


Fig.2 Comparison of the feedback filter tap

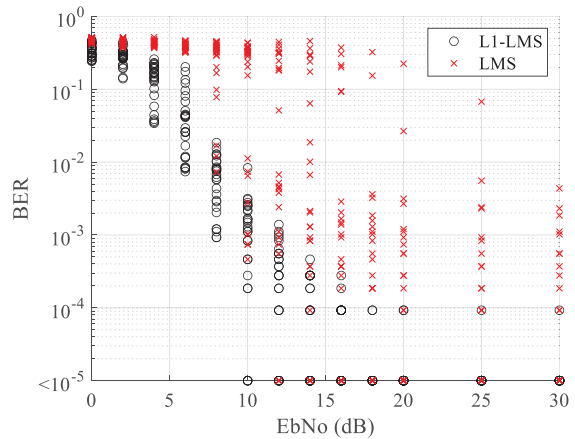


Fig. 3 Comparison of the BER performance