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A method for deriving the effective acoustic impedance of one-dimensional phononic metamaterials

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1. Introduction

Locally resonant acoustic metamaterials with resonators built into individual unit cells can exhibit anomalous behavior while utilizing lattice constants much smaller than the acoustic wavelength. Since the acoustic wavelength is much longer than the lattice constant of locally resonant acoustic metamaterials, macroscopic and effective properties can be assigned to metamaterials by considering scattering in the average sense [1]. The development of these acoustic metamaterials has demonstrated negative acoustic properties such as negative mass density and bulk modulus, expanding the range of material responses found in nature. Negative effective acoustic properties are dynamic and dispersive in nature.

Methods for extracting these effective properties from the experimentally measurable reflection and transmission coefficients were presented, and issues such as metamaterial boundary positions and sign selection of refractive index and impedance were discussed [2]. Here, it should be emphasized that unambiguously determining the sign of the parameters is essential for discussing negative acoustic materials.

We reexamined the method of deriving the effective parameters of one-dimensional phononic crystals, and additionally expressed them as functions of known material parameters. In the present proceedings, we briefly show the results of the former.

2. Theoretical method

A method for retrieving effective material properties of electromagnetic materials was extended to acoustic metamaterials in Ref. [2]. In this method, the effective refractive index and acoustic impedance are obtained from the reflection and transmission coefficients of a plane wave normally incident on a single homogeneous slab.

The reflection coefficient r and transmission coefficient t of a plane wave perpendicularly incident on a layer with density ρ and sound speed v placed between two identical media A with density ρ_A and sound speed v_A are given by the following equations [3].

$$r = \frac{\frac{i}{2}\left(\frac{Z}{Z_A} - \frac{Z_A}{Z}\right)\sin kL}{\cos kL - \frac{i}{2}\left(\frac{Z}{Z_A} + \frac{Z_A}{Z}\right)\sin kL}, \tag{1}$$

$$t = \frac{1}{\cos kL - \frac{i}{2}\left(\frac{Z}{Z_A} + \frac{Z_A}{Z}\right)\sin kL}, \tag{2}$$

where $Z_A=\rho_A v_A$ and $Z=\rho v$ are the acoustic impedance of the media A and the layer, and $k=\omega/v$ is the wavenumber defined in the layer, and ω and L are the frequency and thickness of the layer, respectively.

By solving equations (1) and (2), k and Z can be expressed as functions of transmission and reflection coefficients as

$$kL = \frac{\omega L}{v} = n\frac{\omega L}{v_A} = \cos^{-1}\left(\frac{t^2 - r^2 + 1}{2t}\right), \tag{3}$$

$$\frac{Z}{Z_A} = \frac{i}{\sin kL}\left(\frac{1}{t} - \cos kL - \frac{r}{t}\right) . \tag{4}$$

The effective mass density and sound velocity are then calculated from these. Equation (3) is the same as that derived in Ref. [2], but Eq. (4) is a different expression that does not contain the root. Both impedance and wave number (or refractive index n) are complex functions of complex variables and are multi-valued functions. Therefore, there is a problem of choosing sign and branch number. A suitable method should be used to determine the sign and branch number. Equation (4) has a convenient form for determining signs and branch numbers.

3. Numerical examples and discussions

In this section, we demonstrate the results of applying Eqs. (3) and (4) to a one-dimensional phononic crystal without resonators. For one-dimensional phononic crystals, analytical solutions for the transmission and reflection coefficients are known [3]. The transmission and reflection coefficients of a phononic crystal consisting of GaAs and AlAs are used as input data to calculate Eqs. (3) and (4). The number of periods is assumed to be N=12, i.e., L=12D, where D is the unit period.

Figure 1(a) illustrates the frequency dependence of the effective wavenumber. In the frequency gap, the effective wavenumbers are pure imaginary. The real part of the effective wavenumber corresponds to the dispersion relation obtained by folding the phononic bands into the mini-Brillouin zone corresponding to eight times the period. Moreover, the effective refractive index can also be obtained from the frequency dependence of the effective wavenumber.

Figure 1(b) illustrates the frequency dependence of the effective acoustic impedance. Within the frequency gap, the effective impedance is also given as a pure imaginary number. In the middle of the band, the effective impedance is almost frequency-independent, but exhibits a characteristic behavior at the band edges, i.e., the effective impedance is 0 or infinite, depending on the band edge.

By using Eq. (3), the effective wavenumber (or effective refractive index) and effective impedance can be derived from the transmission coefficient and reflection coefficient. However, this method also requires phase information of the reflection and transmission coefficients as input data. As a next attempt, we expressed the effective parameters as a function of known material parameters. The results will also be shown in the presentation.

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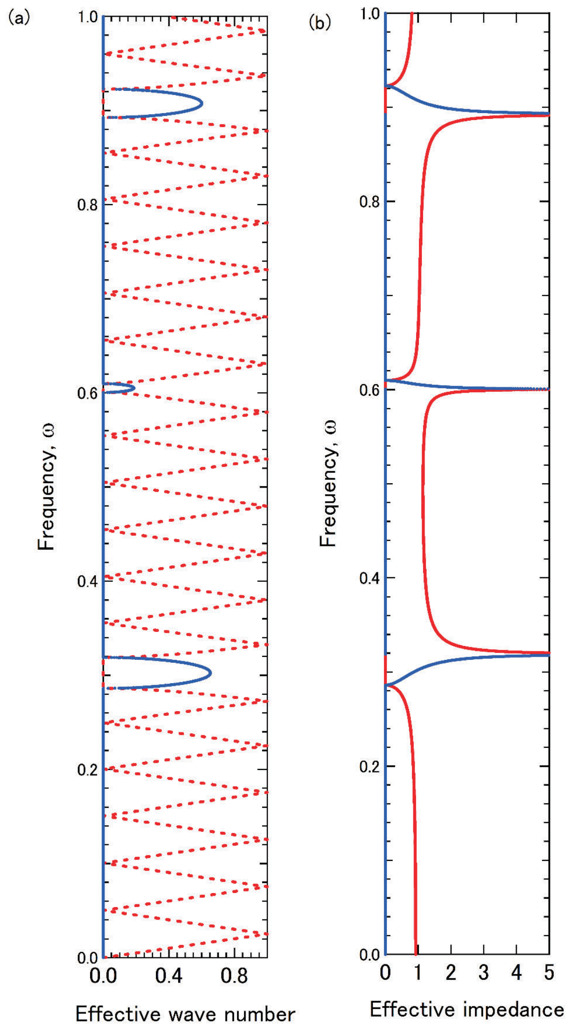

Fig. 1 Effective wave number (a) and effective acoustic impedance (b), which are calculated for a one-dimensional phononic crystal consisting of alternate stacking of GaAs and AlAs layers. The number of bilayers is assumed to be 12. In (a), the dotted lines represent the real part of the effective wave number, whereas the solid lines represent the imaginary part. In (b), red and blue lines represent real and imaginary parts, respectively.

References

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