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A method for measuring small sinusoidal displacement using network analyzer

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1. Introduction

Viscoelasticity of biological tissues is an essential indicator of lesions and their progression. We have proposed a method for estimating viscoelastic parameters by measuring the frequency characteristics of displacement and stress by continuously vibrating an object at an arbitrary frequency with focused ultrasound when an AM-modulated signal is applied to ultrasonic transducers[1)]. However, stable displacement measurement is difficult because the amplitude of displacement generated by ultrasonic excitation decreases (a few µm) as the frequency becomes high.

In the present study, we proposed a method for measuring the amplitude of sinusoidal displacement using a network analyzer to stably measure minute continuous sinusoidal displacements.

2. Theory

2.1. Frequency response of first reflected wave of ultrasonic waves $S_{11}^{(1)}(f)$

As shown in **Fig. 1**, an object at a depth L_0 from a transmitting/receiving ultrasonic transducer is irradiated with ultrasonic waves when (a) the object is stationary and (b) a sinusoidal displacement $\Delta L_{\Delta f}(t) = \Delta L \cdot \sin(2\pi\Delta f t)$ is applied. At this time, the S-parameter $S_{11}^{(1)}(f)$, the frequency f component of the first reflected wave of the received ultrasonic waves, is measured by a network analyzer.

$$S_{11}^{(1)}(f_t) = C(t)^* \cdot y(t), \qquad (1)$$

where $C(t)$ is the transmitted signal, * is the complex conjugate, and $y(t)$ is the received signal. If the one-sided bandwidth of the measurement frequency is ΔB and the center frequency of the measurement frequency band is f_{ctr}, the network analyzer transmits a chirp signal $C(t)$ with a linearly increasing frequency f in the measurement frequency range $[f_{\mathrm{ctr}} - \Delta B : f_{\mathrm{ctr}} + \Delta B]$ and the sweep time $\Delta\tau$.

$$C(t) = \exp(\mathrm{j}2\pi f_t t), \ (f_{\mathrm{ctr}} - \Delta B \le f_t \le f_{\mathrm{ctr}} + \Delta B)$$
$$f_t = (f_{\mathrm{ctr}} - \Delta B) + \frac{2\Delta B}{\Delta\tau} t. \quad (0 \le t \le \Delta\tau) \qquad (2)$$

The sweep frequency f_t of the chirp signal $C(t)$ is a function of time t.

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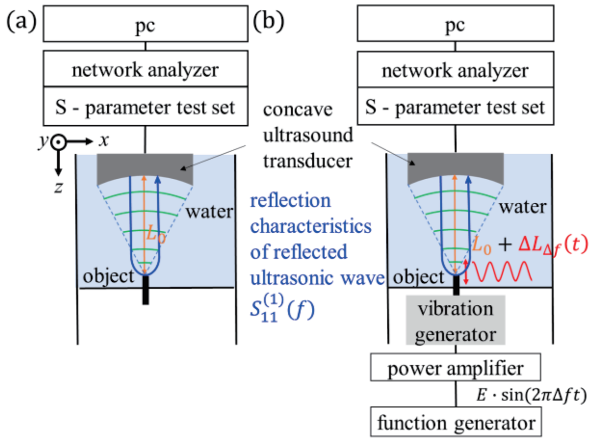

Fig. 1. Experimental system for measuring the amplitude of sinusoidal displacement of an object.

2.2 Estimation method for maximum displacement ΔL of sinusoidal displacement $\Delta L_{\Delta f}(t)$ using inverse Fourier transform $s_{11}^{(1)}(t)$

Consider the complex waveform $y(t)$ of the received signal of the first ultrasonic reflected wave in **Fig. 1(a)**. If the sound velocity in the medium to a depth of L_0 is c, the phase of $y(t)$ is delayed corresponding to the ultrasonic propagation time $\tau_0 = 2L_0/c$ relative to $C(t)$.

On the other hand, in **Fig. 1(b)**, the phase of $y(t)$ is delayed with $\tau_0 + 2\Delta L \cdot \sin(2\pi\Delta f t)/c$ relative to $C(t)$ because the ultrasonic propagation time varies corresponding to the sinusoidal displacement of the object. Therefore, the first reflected wave $S_{11}^{(1)}(f_t)$ of the ultrasonic waves is expressed as

$$S_{11}^{(1)}(f_t) = A_0 \cdot \exp\left\{-\mathrm{j}2\pi f_t \frac{2L_0 + 2\Delta L \cdot \sin(2\pi\Delta f t)}{c}\right\}, \qquad (3)$$

where A_0 is the amplitude characteristic and is to be constant for simplicity. The inverse Fourier transform $s_{11}^{(1)}(t)$ of Eq. (3) is expressed as

$$s_{11}^{(1)}(t) \approx A_0 \left\{\exp(\mathrm{j}2\pi\Delta f b) \cdot j_{-1}\left(t - \tau_0 - \frac{\Delta f \Delta\tau}{2\Delta B}\right) + j_0(t - \tau_0) + \exp(-\mathrm{j}2\pi\Delta f b) \cdot j_1\left(t - \tau_0 + \frac{\Delta f \Delta\tau}{2\Delta B}\right)\right\}, \qquad (4)$$

$$\begin{cases} |j_0(t)| \approx \left|\dfrac{\sin(2\pi\Delta B t)}{2\pi\Delta B t}\right| \\ |j_1(t)| = |j_{-1}(t)| \approx 2\pi \dfrac{\Delta L}{c} f_{\mathrm{ctr}} \cdot \left|\dfrac{\sin(2\pi\Delta B t)}{2\pi\Delta B t}\right| \end{cases}. \qquad (5)$$

$s_{11}^{(1)}(t)$ shows that side peaks occur at $\pm\Delta f \Delta\tau / 2\Delta B$ [s], centered at the peak of $t = \tau_0$ [s].

From Eqs. (4) and (5), the logarithmic amplitude of the side peaks relative to the central peak is expressed as

$$\alpha = 20\log\left(2\pi\frac{\Delta L}{c}f_{\mathrm{ctr}}\right). \quad (6)$$

The maximum displacement ΔL of the object is determined from Eq. (6) as

$$\Delta L = \frac{c}{2\pi f_{\mathrm{ctr}}}10^{\frac{\alpha}{20}}\ \mathrm{[m]}. \quad (7)$$

3. Experiment

Figure 1 shows an experimental system for measuring the amplitude of sinusoidal displacement. First, as shown in **Fig. 1(b)**, a sinusoidal wave $E\sin(2\pi\Delta f t)$ with a voltage amplitude E and a frequency Δf was generated by a function generator, and it was input to a shaker to apply a sinusoidal displacement $\Delta L_{\Delta f}(t) = \Delta L \cdot \sin(2\pi\Delta f t)$ to the object. The displacement frequency Δf was varied from 10 Hz to 60 Hz, and ΔL was varied by the voltage amplitude E. Next, $S_{11}^{(1)}(f_t)$ was measured using a network analyzer, and the maximum displacement ΔL of the object was obtained based on Sect. 2. The displacement measured by the laser displacement meter was used as the reference to evaluate the ultrasound results.

In this measurement, $f_{\mathrm{ctr}} = 0.97$ MHz and $\Delta B = 0.5$ MHz were used, and the sweep time $\Delta\tau$ was changed according to the displacement frequency Δf of the object.

The focusing transducer with a center frequency f_{ctr} of 970 kHz, a focal length of 60 mm, an aperture of 24.6°, and an effective aperture width of 50 mm were used.

4. Results and discussion

Figure 2 shows the measured amplitude characteristic $|s_{11}(t)|$ when the object was applied to a sinusoidal displacement $\Delta L_{\Delta f}(t)$ at $\Delta f =$ 10 Hz. The peak of $s_{11}^{(1)}(t)$ at $t = 82$ μs and side peaks at ± 24 μs from this center peak were observed. This is in good agreement with the theoretical value $\pm\Delta f\Delta\tau/2\Delta B \approx \pm 24.11$ μs based on Eq. (4). The logarithmic amplitude difference α of the side peaks from the center peak was −26.8 dB, and $\Delta L = 11.1$ μm was obtained from Eq. (7). This is in good agreement with the reference of $\Delta L = 10.6$ μm measured with a laser displacement meter. The peak components at $t =$ 164, 246, and 328 μs and their respective side peaks represent the second to fourth multiple ultrasonic reflection components.

Figure 3 shows the measured and reference of the maximum displacement ΔL for each $\Delta f \in$ [10 Hz, 60 Hz] for a sinusoidal displacement $\Delta L_{\Delta f}(t)$. The measured values of ΔL were in good agreement with the references. The average estimation error of sinusoidal displacements of 10 μm or less was 0.4 μm, indicating that very small sinusoidal displacements were well measured. However, the average estimation error for displacements larger than 10 μm was 0.4 μm, resulting in a slightly larger error. This could be due to the approximation accuracy of Eq. (5) in this displacement estimation method. In deriving $s_{11}^{(1)}(t)$, we used the Bessel functions. We assumed that $|-2\pi f_t \cdot \Delta L/c|$ is sufficiently smaller than 1, and obtained $s_{11}^{(1)}(t)$ by using an approximate formula. Therefore, it is considered that the increase in maximum displacement ΔL negatively affected the approximation accuracy of the Bessel functions, then influenced the estimation results of displacement.

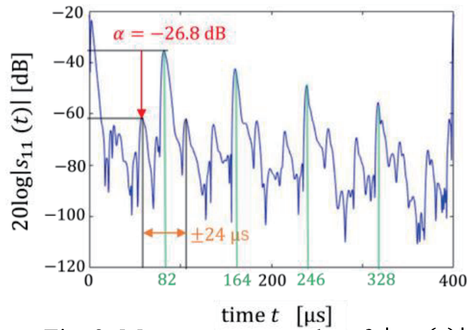

Fig. 2. Measurement results of $|s_{11}(t)|$ when $\Delta L_{\Delta f}(t)(\Delta f = 10\ \mathrm{Hz})$ was applied.

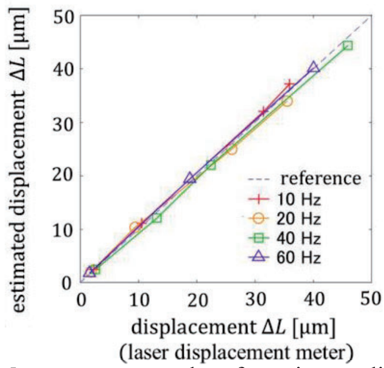

Fig. 3. Measurement results of maximum displacement ΔL of sinusoidal displacement $\Delta L_{\Delta f}(t)$.

5. Conclusion

In this paper, we proposed a method for measuring sinusoidal displacements using a network analyzer with a wide dynamic range and high accuracy of the amplitude and phase for stably measuring the amplitude of small sinusoidal displacements in the order of micrometers. The results showed that the proposed method can measure the amplitude of small sinusoidal displacements of an object vibrating at a frequency Δf. In the future, we aim to measure smaller displacements and higher frequency displacements.

References

1) H. Kawamura et al., Jpn. J. Appl. Phys. **59**, SKKE24, 2020.